

Housing appreciation (depreciation) and owners' welfare: A diagrammatic analysis*

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Abstract. This paper extends Frank's (2006) very simple model to analyze the welfare effects of appreciation and depreciation in a world with property taxes and moving costs. It is shown that appreciation can make homeowners worse off but that depreciation can not make homeowners who intend to stay in their house worse off. Our model provides a simple framework that can be used discuss the rationale for alternative policies to aid homeowners during periods of both appreciation and depreciation.

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1 Introduction

In recent years rapid housing appreciation led to increased residential property tax assessments and widespread citizen protests. In response many states adopted government programs that gave special property tax breaks to homeowners whose parcels had appreciated especially rapidly (see Dye, McMillan and Merriman 2006, and Haveman and Sexton 2008). During the summer and fall of 2008 a rapid and widespread decline in housing prices occurred and there was considerable discussion about appropriate government programs to protect housing equity.

Frank's (2006) wonderfully simple model shows that, starting from a fixed bundle of housing and a composite good, either an increase in the price of housing (appreciation) or a decrease in the price of housing (depreciation) makes an owner better off (pp. 166-7). The appreciation result is intuitive but the case with depreciation is not. This result may seem paradoxical but the intuition behind it is that, once purchased, housing and the composite good become endowments that can be costlessly maintained even when relative prices change.

Frank's logic exercise is an amusing and useful teaching tool but also raises an important and timely question: under what conditions are real world consumers made better (or worse) off by changes in housing prices? This short paper provides some additional intuition about this question by adding two real world complexities-moving costs and property taxes-to Frank's model. The rest of the paper is organized as follows. Section 2 recapitulates Frank's (2006) model, Section 3 revises the model to add moving costs, property taxes are considered in Section 4. Some potential extensions are considered in section 5 and concluding remarks are discussed on Section 6.

2 Frank's Model

Frank's (2006) model can be described graphically as follows (See Figure 1). The consumer has lifetime wealth (w) which she expends on lifetime consumption of h_0 unit of housing (purchased at price p) and $c_0 = w - ph_0$ units of the composite good purchased at a price normalized (without loss of generality) to equal one. The bundle (c_0, h_0) at point A maximizes lifetime utility on indifference curve u_0 .

Suppose housing appreciates after the consumer selects her bundle, then the budget constraint rotates, around the original consumption bundle, to $g'v'$ and intersects u_0 twice. Since the original consumption bundle remains in the opportunity set, the consumer can be no worse off, and will be better off as long as an indifference curve above u_0 is tangent to the new budget constraint along line segment AD .

Now suppose housing depreciates, then the budget constraint rotates, around the original consumption bundle, to $g''v''$ and intersects u_0 twice. Once again the original consumption bundle remains in the opportunity set so the consumer can be no worse off, and will be better off as long as an indifference curve above u_0 is tangent to the new budget constraint along line segment AE .

This simple analysis justifies Frank's (2006) counter-intuitive conclusion that after-the fact changes in prices always increase consumers' well-being in static models. Of course, this surprising result has nothing to do with labeling one axis housing but comes about from the fundamental logic of the model.

3 Moving cost

In Frank's model housing is no different than any other asset. His model suggests that people ought to adjust their housing assets whenever relative prices change but, of course, they do not do this because adjustment of housing assets (by moving, for example) may be costly. In this section we add the assumption that households incur a fixed moving costs when they change houses. How large do moving costs have to be to negate the benefits of appreciation (or depreciation) so that homeowners do not adjust their asset bundles when relative price changes?

We use Figure 2 to answer this question. Figure 2 modifies Figure 1 by depicting only the housing appreciation case and adding a moving cost augmented budget constraint with vertical intercept at $g'' = g' - m$ and horizontal intercept at $v'' = v' - m$, where m is the

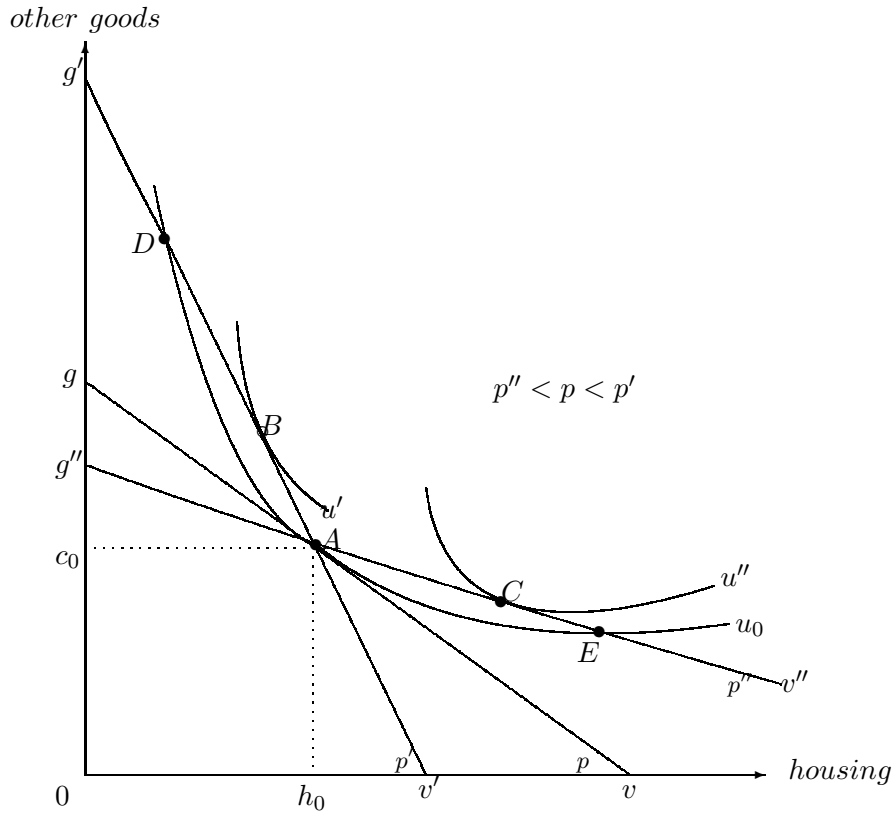


Figure 1: The Frank's (2006) example

constant moving cost. This budget constraint depicts the set of bundles the household can afford after appreciation and paying moving costs. The household will be better off after the price change so long as the moving cost augmented budget is tangent to an indifference curve above u_0 . In Figure 2 we have depicted the breakeven case in which, after appreciation and payment of moving costs, the household can just reach the original indifference curve. In this case, the household is indifferent between consuming the original bundle A or moving to the new bundle H .

Note that the change in housing consumption from bundle A to bundle S is simply the substitution effect from Slutsky's decomposition of a price change while the change in housing from bundle A to bundle H is simply the substitution effect from Hick's decomposition of a price change. Thus, the minimum moving cost such that the household is no better off after a price change is the expenditure required to purchase the Slutsky bundle (S) minus the expenditure required to purchase the Hicksian bundle (H). This difference depends in general

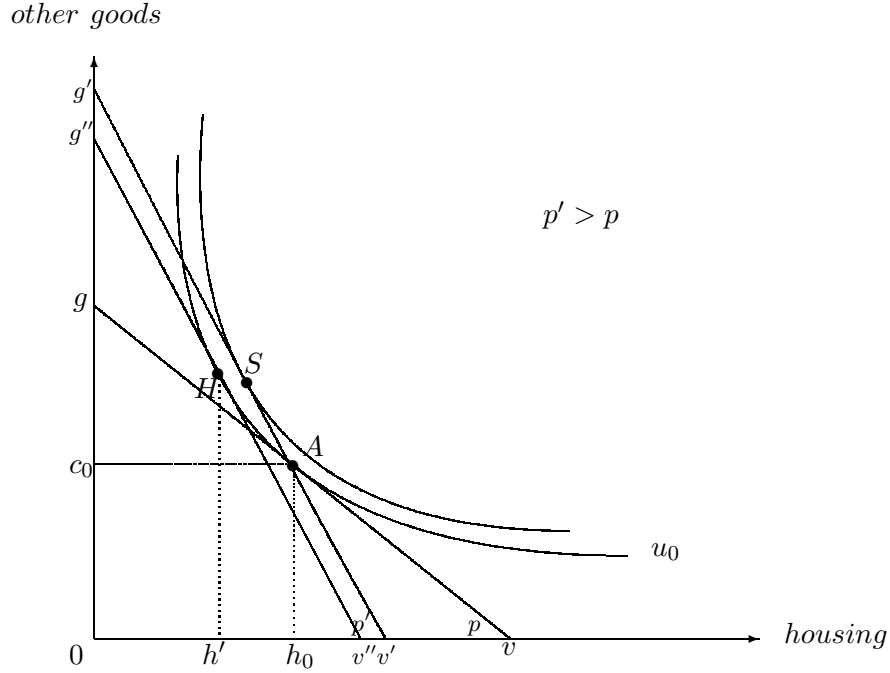


Figure 2: housing appreciation with property tax

on the level of appreciation and the elasticity of substitution in the utility function. The higher the rate of appreciation and the more elastic substitution between the two goods the higher the moving cost consistent with an increase in the household's utility.¹

With moving costs alone appreciation cannot make the household worse off but may prevent the household from taking advantage of a budget reallocation that would otherwise make them

¹Binger and Hoffman (1988) work out some of the relevant mathematics. They show (p.189-193) that for a simple Cobb-Douglas utility function ($U = hc$) the expenditure required to purchase the Hicksian bundle (H in our Figure 2) is simply $w\sqrt{\frac{p'}{p}}$, where p' is the post appreciation price of housing. The expenditure required to purchase the Hicksian bundle increases with the square root of appreciation. The cost of purchasing the Slutsky bundle (S) is the cost of the original bundle (w) plus the potential capital gain (if the household sold all the housing in its original bundle) of $(p' - p)h_0$. In the simple Cobb Douglas case, the cost of the Slutsky bundle minus the cost of the Hicksian bundle is $w[\frac{1}{2}(\frac{p'}{p} + 1) - \sqrt{\frac{p'}{p}}]$. So for example, if the rate of housing appreciation $((p'/p) - 1)$ was ten percent, the household's utility increases as long as moving costs are less than 0.12% of lifetime income. If appreciation rose to 20 percent household utility increase as long as moving costs are less than 0.45% of lifetime income.

better off. The analysis with depreciation is parallel. Again moving costs inhibit mobility and prevent the household from reallocating its budget to increase utility.

4 Property taxes

In the US (and many other countries) housing is different from virtually all other assets in an important way, because owners are assessed a “property tax” that is roughly proportional to the market value of the asset. In order to more fully understand the welfare implications of housing appreciation we add a property tax to Frank’s model.

We first consider the case with property taxes but no moving costs. Suppose initially that the property tax inclusive relative price of housing² is $p(1+t) = (g/v)$ and that a household consumes a bundle consisting of h_0 units of housing and c_0 units of the composite good and attains utility level u_0 (point A , Figure 3). We assume that our graph depicts a small neighborhood in a large community so that the tax revenue collected in this neighborhood has a negligible effect on overall tax revenue and public services when there is appreciation or depreciation.

Now imagine that we have real estate appreciation so that the relative (property tax inclusive) price of housing rises to $p'(1+t) = (g'/v')$. Note that the new budget constraint ($g'v'$) does not rotate around bundle A because with appreciation comes an increased property tax liability which means the homeowner can no longer afford her original bundle (h_0, c_0) . If the homeowner were to stay in the same home consumption of the other goods would have to drop from c_0 to c' (bundle A') so that the owner could pay her property taxes.³ As we have drawn Figure 3, the owner’s new (post appreciation) budget constraint is tangent to indifference curve u_0 at bundle B . Thus, as depicted, the owner is just as well off after appreciation (i.e. with budget constraint $g'v'$) as she was before. Because of the appreciation (and rise in property tax liability) the owner chooses to move to a smaller house (h') but consume more of the composite good.

Although we have depicted the breakeven case in Figure 3, it should be clear that, once property taxes are considered, appreciation can make the owner better off (as in Frank’s model)

²In our static model households make a lifetime allocation of wealth to housing or non-housing goods. In this context, the lifetime (rather than annual) cost of property taxes is relevant. The lifetime cost of the property tax is the present value of all future payments.

³If bundle A' contains less than the minimum acceptable level of non-housing goods (c) (which could be zero, or a minimum level required for survival) we might deem the homeowner “liquidity constrained”. Such a homeowner would be forced to move (and incur moving costs) in order to pay property taxes.

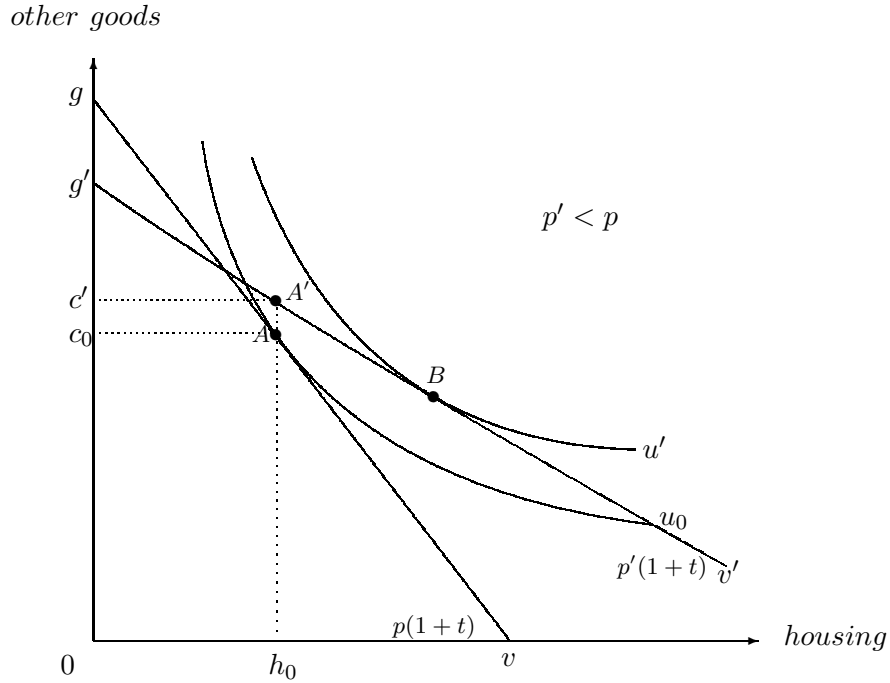


Figure 4: Housing depreciation with property taxes

those that own homes since, even if they stay in the same house their property tax payments fall. If the total value of moving costs and increased property tax payments in the original home exceeds the difference in expenditure needed to purchase the Slutsky and Hicksian bundles the homeowner will be worse off after appreciation.

5 Potential extensions

Our analysis uses a very simple, stylized and static model to gain insight about basic issues. In this short section we provide informal analysis of three extensions that might be considered explicitly in more general models.

We assume that households make a single lifetime allocation of wealth to housing and non-housing consumption. In fact, people often plan to adjust housing consumption as family size grows and ebbs during the life course. Households that planned to move to smaller houses

in any case, would benefit more from appreciation and less from depreciation than shown in our results.

Secondly, we assume uniform housing appreciation (depreciation) and constant property tax rates. However, if housing prices increase (decrease) government can cut (must increase) the property tax rate to maintain revenue. If housing price changes are uniform and government changes the property tax rate to keep revenue constant we need only worry about moving costs when assessing the impact of housing price changes on households' well-being. If appreciation (depreciation) is not uniform our analysis could be applied to households that own units that undergo relative appreciation or depreciation.

Finally, our static model does not explicitly depict the utility that homeowners get from bequests. If households care about the consumption stream coming from their housing bequest our omission does not alter the analysis since housing simply enters the utility function twice, once as a consumption good, and once as a bequest. However, if households care about the cash value of their bequest then the monetary value of their housing asset enters the utility function directly. In this case, appreciation would be more likely to improve the welfare of homeowners and depreciation could make homeowners worse off.

6 Conclusion

Frank (2006) provides a paradoxical example that demonstrates that housing appreciation and depreciation both benefit owners. We add more structure to the model by considering moving costs and property taxes and show that appreciation can make homeowners either better or worse off, depending on the relative size of the moving costs and the benefits from altering consumption bundles when relative prices change. On the other hand, whether the homeowner moves or stays depreciation must make her better off when there is a property tax. These results could be used to partially justify some of the recently enacted government programs to mitigate increases in property tax payments resulting from rapid housing appreciation on equity grounds, especially for households that have high moving costs. Of course, the total effect of these programs could still be negative since recent studies (Haveman and Sexton 2008) find that they often have unwanted efficiency or equity effects.

References

- [1] Binger, Brian R. and Elizabeth Hoffman, (1988), *Microeconomics with Calculus*. Glenview, Illinois: Scott, Foresman and Company.
- [2] Dye, Richard, Daniel McMillen, and David Merriman, (2006), "Illinois' Response to Rising Residential Property Values: An Assessment Growth Cap in Cook County," *National Tax Journal* 59(3), 707-716.
- [3] Frank, Robert. (2006), *Microeconomics and Behavior*, 6th ed. McGraw-Hill Companies, Inc., New York.
- [4] Haveman, Mark and Terri A. Sexton, (2008), *Property Tax Assessment Limits: Lessons from Thirty Years of Experience*, Cambridge, Mass.: Lincoln Institute of Land Policy.